

# Collective modes of the $d$ -density wave state and its relevance to high- $T_c$ cuprates

Jay D. Sau<sup>1</sup>, Ipsita Mandal<sup>2</sup>, Sumanta Tewari<sup>3</sup>, and Sudip Chakravarty<sup>2</sup>

<sup>1</sup>*Department of Physics, Harvard University, Cambridge, MA 02138*

<sup>2</sup>*Department of Physics and Astronomy, University of California Los Angeles, Los Angeles, California 90095-1547*

<sup>3</sup>*Department of Physics and Astronomy, Clemson University, Clemson, SC 29634*

We calculate the collective mode spectra of the  $d$ -density wave state, proposed to describe the anomalous phenomenology of the cuprates in the pseudogap regime. Even though the state breaks translational symmetry by a lattice spacing and is described by a particle-hole singlet order parameter condensing at the wave vector  $q = Q = (\pi, \pi)$ , interestingly, we find that the amplitude collective mode spectrum can have inelastic peaks at both  $q = (0, 0)$  and  $q = Q = (\pi, \pi)$ . In general, the spectra is non-universal, and, depending on the microscopic parameters, can have one or two peaks in the two-dimensional Brillouin zone, signifying confluence of two kinds of magnetic excitations. Our theory should be important in interpreting inelastic neutron scattering experiments in high- $T_c$  cuprates, where such multiple magnetic excitations are implicated.

PACS numbers: 74.20.-z, 74.25.Dw, 71.45.-d

Ever since the discovery of pseudogap in high temperature superconductors it has been a profound mystery [1]. To this day, its origin is vigorously debated. In one view, pseudogap is a remnant of the  $d$ -wave superconducting gap that defines a crossover temperature  $T^*$  in the phase diagram. The other view argues for a broken symmetry at  $T^*$ . The precise nature of the broken symmetry is debated, however [2–4]. Here, we shall assume that much of the phenomenology associated with the pseudogap can be described in a unified manner by the single assumption of a spin singlet  $d_{x^2-y^2}$  density wave (DDW) [3]. The purpose of the present Letter is to deduce certain observable consequences of this assumption, namely multiple collective modes resulting from this ordered state, which can possibly arise in inelastic neutron scattering experiments, as in Refs. [5, 6]. As we shall see, these turn out to be unusual and were missed in a very early paper [7] on this subject. As to elastic signature of singlet DDW, two neutron scattering measurements seem to provide some evidence for it [8]. However, these measurements have not been confirmed by further independent experiments.

The singlet DDW, as originally envisioned, consists of circulating currents [9], alternating between clockwise and anti-clockwise directions in the neighboring plaquettes of an underlying square lattice in two-dimensional (2D)  $\text{CuO}_2$  planes. Viewed from this perspective, the state can be constructed as a juxtaposition of two kinds of current conserving vertices, as shown in vertices 5 and 6 of Fig. 1. The ordered state breaks time reversal, rotation by  $\pi/2$ , parity, and translational symmetry by one lattice spacing, but the product of any two of these symmetries is preserved. Superficially, the statistical mechanics belongs to the Ising universality class [9], and in mean field theory the order disappears when the magnitude of the bond currents vanishes with increase of temperature. However, fluctuations can reverse an arrow if it is possible to do so in a current conserving manner. This can be done by enlarging the configuration space by adding four additional vertices shown as vertices 1, 2, 3, and 4 in Fig. 1 [10]. The vertices corresponding to sources and sinks should have a large negative chemical potential. The model then belongs to the

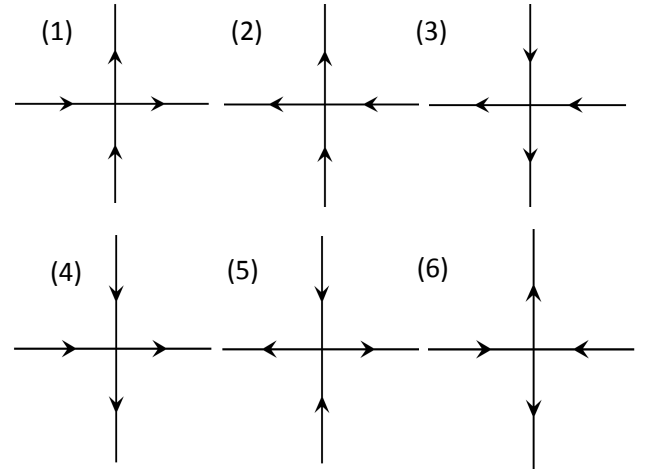


FIG. 1. The six possible current vertices in the 6-vertex model. The vertices (5) and (6) are the AF vertices which lead to the DDW phase with local orbital moments, while the rest of the vertices lead to longer range current fluctuations.

universality class of the classical six-vertex model [10]. The quantum six-vertex model and its quantum criticality has also been previously discussed [11].

The particle-hole spin-singlet DDW order parameter is

$$\langle c_{\mathbf{k},\alpha}^\dagger c_{\mathbf{k}+\mathbf{Q},\beta} \rangle = i\Delta_Q f_{\mathbf{k}} \delta_{\alpha\beta}, \quad (1)$$

where  $c, c^\dagger$  are electron annihilation and creation operators, and  $\alpha, \beta$  are the spin indices;  $\Delta_Q$  is the magnitude of the order parameter and the form factor  $f_{\mathbf{k}} = (\cos k_x - \cos k_y)$ ; we set the lattice constant to unity throughout. Because the order parameter condenses at the wave vector  $Q = (\pi, \pi)$ , one would have naively expected that the amplitude collective mode, the analog of the  $U(1)$  Anderson-Higgs mode, but in the particle-hole channel, would be peaked at the same wave vector.

An important result of our paper is that the spectra of the amplitude fluctuations of the DDW state are *not* confined to

only the ordering wave vector  $Q$ , but can have finite-frequency peaks at both  $q = (0, 0) \equiv 0$  and  $q = Q$ , as well as considerable spectral weight over a substantial region of the momentum space. The emergence of a  $q = 0$  peak, even with a mean field state that breaks the lattice translation symmetry such as the twofold commensurate DDW, indicates  $q = 0$  fluctuations hidden within the DDW state. We find that these fluctuations reflect an *orbital ferromagnetic* (OF) phase in the six-vertex generalization of the model, as mentioned above. In its phase diagram one finds OF, as well as orbital antiferromagnetic (DDW) phases [10]. The existence of such striking  $q = 0$  fluctuations, even in the absence of an OF order parameter, can be important in interpreting inelastic neutron scattering experiments [5]. However, because the response is at higher frequencies, they are subject to considerable degree of non-universality and serves as a warning that if our theory is correct, experimental signatures should not be unique across materials, as well as within a given material, as a function of doping, frequency, etc.

To calculate the DDW collective mode spectrum we start with the electronic Hamiltonian which has the form,

$$H = \sum_{k,\sigma} (e_k - \mu) c_{k,\sigma}^\dagger c_{k,\sigma} + g^{-1} \sum_q \hat{\Delta}_q^\dagger \hat{\Delta}_q, \quad (2)$$

where

$$\hat{\Delta}_q = \frac{g}{2} \sum_k f_{k+(q-Q)/2} c_k^\dagger c_{k+q}, \quad (3)$$

Here we adopt a commonly used dispersion  $e_k$  based on local density approximation [12], which is

$$e_k = -2t(\cos k_x + \cos k_y) + 4t' \cos k_x \cos k_y - 2t''(\cos 2k_x + \cos 2k_y). \quad (4)$$

For subsequent notational simplicity, we define the functions,  $\varepsilon_k = \frac{1}{2}(e_k + e_{k+Q})$ , and  $\epsilon_k = \frac{1}{2}(e_k - e_{k+Q})$ . The band parameters are chosen to be  $t = 0.15$  eV,  $t' = 0.3t$ , and  $t'' = 0.5t'$ . The difference with the conventional LDA band structure is a rough renormalization of  $t$  (from 0.38 eV to 0.15 eV), which is supported by experiments involving angle resolved photoemission spectroscopy [13]. Note that in Eq. 3, as elsewhere, we shall drop the spin-index  $\sigma$ , as it will play no important role.

For an analysis of the collective modes of the DDW state at a general  $q$ , we have generalized the gap parameter in Eq. 3. In this form,  $\hat{\Delta}_q^\dagger = \hat{\Delta}_{-q}$ , for an arbitrary  $q$ , which implies that the gap parameter is purely real in the real space. This constraint is necessary to ensure that no gapless phase mode is generated in the collective mode spectrum of the twofold commensurate DDW state which breaks only discrete symmetries [7]. From Eq. 3 it follows that at  $q = Q$ ,  $\hat{\Delta}_Q$  represents the conventional DDW gap parameter shown in Fig. 1 in its vertex representation, while  $q = 0$  leads to a gap parameter  $\hat{\Delta}_0$  which represents uniform current flow along the  $+x$  and  $+y$  directions. Choosing  $q = Q = Q + \bar{Q}$ , where  $\bar{Q} = (\pi, -\pi)$ , we find currents flowing along the  $+x$  and  $-y$

directions. These, together with reversal of currents, give the remaining vertices in Fig. 1. Finally, at  $q = \bar{Q}$ ,  $\hat{\Delta}_q$  allows a gap parameter that breaks local current conservation with sites having a current source at one vertex and a current sink in the neighboring vertex. However, in this paper we will choose the fugacity of these current-conservation violating vertices, which are controlled by the coupling constant  $g$  in front of the form factors, to vanish [10]. Thus the operator  $\hat{\Delta}_q$  represents the full set of current vertices of the six-vertex model. We find that only the DDW gap parameter  $\hat{\Delta}_Q$  develops a mean-field expectation value in the saddle-point solution, though we will find fluctuations from all the other vertices as well.

Anticipating a twofold commensurate DDW order with the order parameter given by Eq. (1), we first fold the full 2D Brillouin zone (BZ) to the reduced BZ (RBZ). The reduced zone is defined in terms of the rotated coordinates,  $(k_x + k_y)/\sqrt{2} = k'_x, (k_x - k_y)/\sqrt{2} = k'_y$ , so that in the RBZ  $k'_x, k'_y \in [-\pi/\sqrt{2}, \pi/\sqrt{2}]$ . Note that the Dirac points in the spectrum occur at  $(k'_x, k'_y) = (\pi/\sqrt{2}, 0)$  and  $(k'_x, k'_y) = (0, \pi/\sqrt{2})$ . The vector  $Q$  is at  $(\sqrt{2}\pi, 0)$ , while  $\bar{Q} = (\pi, -\pi)$  in the original basis is now at  $(0, \sqrt{2}\pi)$ .

To facilitate our discussion, we introduce the spinor notation  $\hat{\Psi}_k^\dagger = (c_k^\dagger, c_{k+Q}^\dagger)$ . Defining a BZ periodic function  $u_k$  which is 1 inside the RBZ and zero outside,  $\hat{\Delta}_q$  may be written as  $(\sigma_i, i = 1, 2, 3$  are the conventional Pauli matrices)

$$\hat{\Delta}_q = \sum_k u_k f_{k+(q-Q)/2} \Psi_k^\dagger [u_{k+q} \sigma_3 + i u_{k+q+Q} \sigma_2] \Psi_{k+q} \quad (5)$$

The above expression for  $\hat{\Delta}_q$  is defined for  $q$  in the full BZ. In the RBZ this corresponds to four different gap parameters. It is convenient to split  $\hat{\Delta}_q$  as

$$\begin{aligned} \hat{\Delta}_q &= \sum_k u_k u_{k+q} f_{k-Q/2+q/2} \Psi_k^\dagger \sigma_3 \Psi_{k+q} \\ \hat{\Delta}_{q+Q} &= \sum_k u_k u_{k+q} f_{k+q/2} \Psi_k^\dagger \sigma_2 \Psi_{k+q} \\ \hat{\Delta}_{q+Q} &= \sum_k u_k u_{k+q} f_{k-\bar{Q}/2+q/2} \Psi_k^\dagger \sigma_3 \Psi_{k+q} \\ \hat{\Delta}_{q+Q+Q} &= \sum_k u_k u_{k+q} f_{k+q/2+Q/2} \Psi_k^\dagger \sigma_2 \Psi_{k+q}. \end{aligned} \quad (6)$$

where  $Q = (Q + \bar{Q})$  and we have dropped the imaginary  $i$  in  $\hat{\Delta}_{q+Q}$  and  $\hat{\Delta}_{q+Q+Q}$ . After this transformation, the vector  $q$  only takes values which are the differences of the wave-vectors in the RBZ. Physically, these 4 operators (with positive and negative signs) now correspond to the 8 current orderings of the eight-vertex model. As discussed above, we will choose the  $g$  associated with the non-current conserving vertices to vanish, thus leaving us with a six-vertex model. The phase transition of the six-vertex model does not have a singular specific heat at its transition, whereas the eight vertex model does; this is in accord with experiments in high temperature superconductors.

For compactness of notation we will write  $\hat{\Delta}_{b,q}^{(a)} =$

$\hat{\Delta}_{q+b+aQ}$ , where  $a = 0, 1$  and  $b = 0, Q$ . With this notation, the hermiticity condition on  $\hat{\Delta}_q$  translates into the condition  $\hat{\Delta}_{b,q}^{\dagger(a)} = \hat{\Delta}_{b,-q}^{(a)}$ . Furthermore it is convenient to write  $\hat{\Delta}_{b,q}^{(a)} = \sum_k \Psi_{k+q}^\dagger \rho_{b,k+q/2}^{(a)} \Psi_k$ , where  $\rho_{b,k+q/2}^{(a)}$  are  $2 \times 2$  matrix-valued structure factors. The Hamiltonian in the spinor notation is

$$H = \sum_k \Psi_k^\dagger ((-\mu + \varepsilon_k) \sigma_0 + \varepsilon_k \sigma_3) \Psi_k + g^{-1} \sum_{q,a,b} [\hat{\Delta}_{b,q}^{(a)\dagger} \hat{\Delta}_{b,q}^{(a)}], \quad (7)$$

where in the sums over  $a$  and  $b$ ,  $a = 1$  and  $b = Q$  are dropped because these vertices do not conserve currents. Consider now the imaginary-time effective action and decouple the four-fermion term  $\sum_{q,a,b} \hat{\Delta}_{b,q}^{(a)\dagger} \hat{\Delta}_{b,q}^{(a)}$  in Eq. 7 using the Hubbard-Stratonovich transformation. This leads to the effective action,

$$S = \sum_{k,\omega} \Psi_{k,\omega}^\dagger ((-i\omega - \mu + \varepsilon_k) \sigma_0 + \varepsilon_k \sigma_3) \Psi_{k,\omega} - 2g^{-1} \sum_{q,\omega,a,b} \hat{\Delta}_{b,q,\omega}^{(a)\dagger} \Delta_{b,q,\omega}^{(a)} + g^{-1} \sum_{q,\omega,a,b} \Delta_{b,q,\omega}^{*(a)} \Delta_{b,q,\omega}^{(a)}, \quad (8)$$

where the Hubbard-Stratonovich fields satisfy the constraint  $\Delta_{b,-q,-\omega}^{(a)} = \Delta_{b,q,\omega}^{*(a)}$ . Expressing the  $\hat{\Delta}$  operators in Eq. (8) in terms of the fermion spinors  $\Psi$ 's and then performing the Grassmannian path integral over the quadratic terms leads to the effective action,

$$S = g^{-1} \sum_{q,\omega,a,b} \Delta_{b,q,\omega}^{*(a)} \Delta_{b,q,\omega}^{(a)} - \text{Tr} [\ln(M_{0,k_1,\omega_1} \delta_{k_1,k_2} \delta_{\omega_1,\omega_2} + \delta M_{\omega_1,\omega_2}^{k_1,k_2})], \quad (9)$$

where

$$M_{0,k,\omega} = (-i\omega - \mu + \varepsilon_k) \sigma_0 + \varepsilon_k \sigma_3 - \sum_{a,b} \rho_{b,k}^{(a)} \Delta_{b,0}^{(a)}, \quad (10)$$

$$\delta M_{\omega_1,\omega_2}^{k_1,k_2} = -u_{k_1} u_{k_2} \sum_{a,b} \rho_{b,(k_1+k_2)/2}^{(a)} \Delta_{b,k_1-k_2,\omega_1-\omega_2}^{(a)}. \quad (11)$$

The mean-field equation for the DDW state is obtained by setting the terms which are linear in  $\delta M$  in Eq. 9 to zero. This leads to the equation,

$$\frac{1}{g} = \sum_k \frac{f_k^2}{E_k} [n_F(\frac{\varepsilon_k + E_k - \mu}{2T}) - n_F(\frac{\varepsilon_k - E_k - \mu}{2T})]. \quad (12)$$

for the order parameter  $\Delta_Q$ ;  $E_k = \sqrt{\varepsilon_k^2 + f_k^2 \Delta_Q^2}$ . and  $n_F(E)$  is the Fermi-function. The chemical potential  $\mu$  is determined from the hole-doping  $x$ . There are no static saddle point solutions corresponding to  $\langle \hat{\Delta}_q \rangle$  and  $\langle \hat{\Delta}_{q+Q} \rangle$  for the range of parameters considered. However, interestingly, the collective mode spectrum for the DDW state will contain a component also near  $q \sim 0$ , which can be interpreted as orbital ferromagnetic fluctuations of the bond currents.

The collective modes are calculated by considering the terms second order in  $\Delta_{q,\omega}$  in Eq. 9, which have the form

$$\delta S^{(2)} = \frac{1}{2} \sum_{k_1,\omega_1} \text{Tr} [u_{k_1} u_{k_1+q} \{ \sum_{a,b} \rho_{b,k_1+q/2}^{(a)} \Delta_{b,q,\omega_1-\omega_2}^{(a)*} \} M_{0,k_1+q,\omega_1+\omega}^{-1} \{ \sum_{a,b} \rho_{b,k_1+q/2}^{(a)} \Delta_{b,q,\omega_1-\omega_2}^{(a)} \} M_{0,k_1,\omega_1}^{-1}] + g^{-1} \sum_{\omega,a,b} \Delta_{b,q,\omega}^{*(a)} \Delta_{b,q,\omega}^{(a)}, \quad (13)$$

Performing the Matsubara summations and analytically continuing to real frequency leads to

$$\delta S^{(2)} = \sum_{a,b,a',b'} \Delta_{b,q,\omega}^{(a)*} U_{bb'}^{(a,a')}(q, \omega) \Delta_{b,q,\omega}^{(a)}, \quad (14)$$

where  $U_{bb'}^{(a,a')}(q, \omega)$  are given by

$$U_{bb'}^{(a,a')}(q, \omega) = \sum_{k_1,m,n} \Lambda_{m,n}(k_1, k_1+q; \omega) \text{Tr} (\rho_{b,k_1+q/2}^{(a)} A_{m,k_1} \rho_{b',k_1+q/2}^{(a')} A_{n,k_1+q}) + g^{-1} \delta_{a,a'} \delta_{b,b'} \quad (15)$$

and

$$\Lambda_{m,n}(k_1, k_1+q; \omega) = - \frac{n_F(mE_{k_1} + \varepsilon_{k_1} - \mu) - n_F(nE_{k_1+q} + \varepsilon_{k_1+q} - \mu)}{2(\omega - mE_{k_1} + nE_{k_1+q} - \varepsilon_{k_1} + \varepsilon_{k_1+q} + i\delta)}, \quad (16)$$

with  $A_{m,k} = \frac{1}{2E_k} (E_k + m\varepsilon_k \sigma_3 - m f_k \Delta_Q \sigma_2)$ . For the rest of the paper we will focus on the spectra at the special  $q$  points  $q = Q$  and  $q = 0$  where the  $U$  coefficients take the form

$$U_{Q,Q}^{(a,a)}(0, \omega) = \sum_{k: E_k > |\mu - \varepsilon_k|} \frac{\omega^2 f_k^2 - 4\Delta_Q^2 f_k^4 - 4\varepsilon_k^2 (f_k^2 - f_{Q,k}^{(a)2})}{E_k(\omega^2 - 4E_k^2)}, \quad (17)$$

and

$$U_{0,0}^{(a,a)}(0, \omega) = \sum_{k: E_k > |\mu - \varepsilon_k|} f_k^2 \frac{\omega^2 - 4\varepsilon_k^2 - 4\Delta_Q^2 (f_k^2 - f_{0,k}^{(a)2})}{E_k(\omega^2 - 4E_k^2)} \quad (18)$$

respectively at  $T = 0$ . In this limit, we find that all terms, where  $b \neq b'$  or  $a \neq a'$ , vanish because of the different symmetries of the form factors under  $k_x \rightarrow -k_x$  and  $k_y \rightarrow -k_y$ .

The measurement of dissipation from any probe that couples to the fields  $\hat{\Delta}_q$  and  $\hat{\Delta}_{q+Q}$  would be of the form

$$\Im[\langle \Delta_{b,q,\omega}^{(a)} \Delta_{b',-q,-\omega}^{(a')} \rangle] = \Im[V_{b,b'}^{(a,a')}(q, \omega)] \quad (19)$$

with

$$V(q, \omega) = \left( U_{b,b'}^{(a,a')}(q, \omega) \right)^{-1}, \quad (20)$$

where  $(a, b)$  and  $(a', b')$  are treated as matrix indices. Thus the peaks in the imaginary part of the spectrum near  $q = 0$  and  $Q$  are given by the zeroes of the functions in Eq. 17 and Eq. 18. The  $a = 0$  (corresponding to the conventional DDW order parameter) propagator near  $Q$ , given by Eq. 17, can be easily seen to vanish when  $\omega \sim 2\Delta_Q$  because of the numerator. On the other hand, low  $\omega$  zeroes in the other propagators only emerge when  $\mu$  is such that the denominator  $(\omega^2 - 4E_k^2)$  can vanish while satisfying the condition  $E_k > |\mu - \varepsilon_k|$ . This occurs in the presence of a non-zero next nearest neighbor coupling  $t'$ , which breaks particle-hole symmetry in the band structure.

There is no mean field OF order coexisting with the twofold commensurate DDW order in the regime of doping studied here. In contrast, from Fig. 2 it is clear that in the underdoped regime there is a noticeable finite-frequency peak at  $q = 0$  coexisting with a finite frequency peak at  $Q$ . The intensity at  $q = 0$  goes down with increasing  $x$ , but the amplitudes strongly depend on the microscopic parameters, which implies that the amplitude fluctuation spectra can have finite frequency peaks at both wave vectors  $q = 0$  and  $q = Q$ , or only a single peak. Such non-universality indicates that different families of cuprates, or even samples at different hole doping within the same family of cuprates, may have different fluctuation spectra. The non-universality of the collective modes and the coexistence of finite frequency peaks at multiple wave vectors are the central aspects of our work. These results are direct consequences of the vertex model framework of the fluctuations for the DDW state and may have important consequences for the inelastic neutron scattering experiments, which are left for future study.

In summary, we have calculated the collective mode spectra for the commensurate DDW state, which was proposed earlier to explain the phenomenology of the pseudogap phase of the high- $T_c$  cuprates. To go beyond the fluctuations at  $Q$ , the DDW gap parameter was generalized as in Eq. 3. The persistence of a hidden,  $q = 0$  OF fluctuations even when the ground state is the twofold commensurate DDW state is natural from the vertex model description of the DDW state. In principle, one can also expect fluctuations associated with the  $q = 0$  nematic state, which, as shown in Ref. [14], is comparable in energy to the DDW state. This nematic order parameter  $\Delta_{\text{nem}} = \sum_k f_k c_k^\dagger c_k$  is a close cousin of the order parameter discussed here. Finally, because the orbital magnetic moment contribution to inelastic neutron scattering is expected to be enhanced at long wavelengths, the relatively small peak at  $q = 0$  can produce scattering peaks comparable to that at  $Q$ . However, the detailed kinematics addressing quantitatively the neutron scattering signatures of the collective modes are beyond the scope of the present paper. In any event, because of the non-universality of the inelastic spectra, as mentioned above, such an effort will not be particularly meaningful at

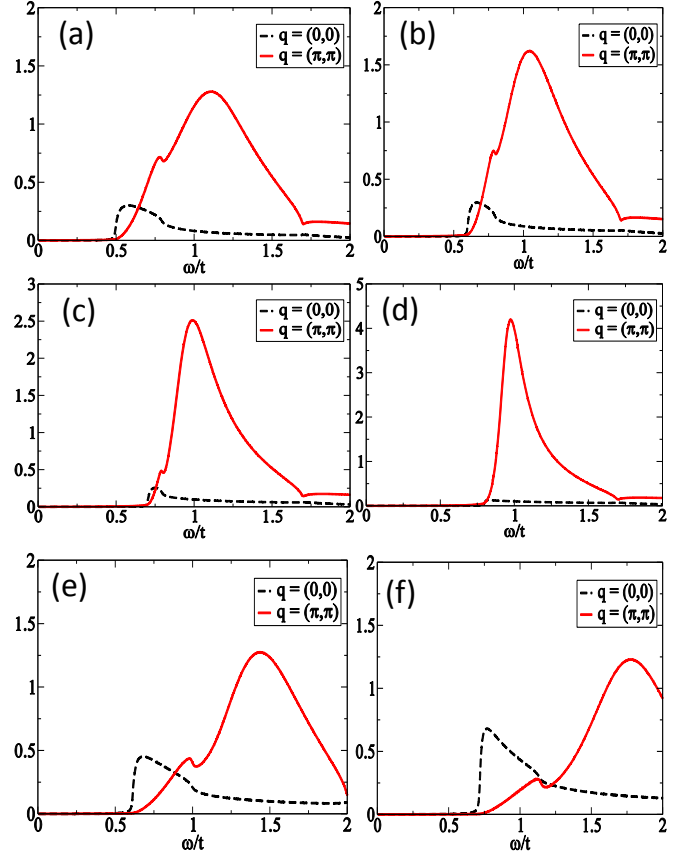


FIG. 2. (Color online) Collective mode response of the commensurate DDW state as a function of  $\omega/t$  for  $q = Q$  (solid curve) and  $q = 0$  (dashed curve) for the band parameters given in the text. In the panels (a)-(c),  $\Delta_Q = 0.3t$ , and  $x = 0.06$  (a),  $x = 0.1$  (b),  $x = 0.14$  (c),  $x = 0.18$  (d). The emergence of an inelastic  $q = 0$  fluctuation peak with underdoping is a signature of hidden underlying OF fluctuations co-existing with the commensurate DDW fluctuations centered at  $q = Q$ . The vertical axes correspond to arbitrary units, but are the same for all panels. The panels (e) and (f) correspond to  $x = 0.06$  and  $\Delta_Q = 0.4t$  and  $\Delta_Q = 0.5t$  respectively.

this time.

J. D. S acknowledges support from the Harvard Quantum Optics Center. I. M. was funded by David. S. Saxon Presidential Term Chair at UCLA. S. T. would like to thank DARPA-MTO, Grant No. FA9550-10-1-0497 and NSF, Grant No. PHY-1104527 for support. S. C. was supported by a grant from NSF, Grant No. DMR-1004520. J. D. S. and S. C. also acknowledge support from NSF Grant No. PHY-1066293 and the hospitality of the Aspen Center for Physics where the work was completed. We would also like to thank B. Keimer, M. Greven and H. A. Mook for discussions regarding their published and unpublished works.

- [2] C. M. Varma, Phys. Rev. B **55**, 14554 (1997); *ibid* **75**, 15113 (2006).
- [3] S. Chakravarty, R. B. Laughlin, D. K. Morr, C. Nayak, Phys. Rev. B **63**, 094503 (2001).
- [4] S. A. Kivelson *et al.* Rev. Mod. Phys. **75**, 1201 (2003).
- [5] Y. Li *et al.* Nat. Phys. **8**, 404 (2012).
- [6] H. A. Mook, M. B. Stone, J. W. Lynn, M. B. Lamsden, and A. D. Chistianson, unpublished.
- [7] S. Tewari and S. Chakravarty, Phys. Rev. B **66**, 054510 (2002).
- [8] H. A. Mook *et al.* Phys. Rev. B **66**, 144513 (2002). H. A. Mook *et al.* Phys. Rev. B **69**, 134509 (2004).
- [9] C. Nayak, Phys. Rev. B **62**, 4880 (2000).
- [10] S. Chakravarty, Phys. Rev. B **66**, 224505 (2002).
- [11] O. F. Syljuasen and S. Chakravarty, Phys. Rev. Lett. **96**, 147004 (2006)
- [12] E. Pavarini *et al.* Phys. Rev. Lett. **87**, 047003 (2001).
- [13] A. Damascelli, Z. Hussain, and Zhi-Xun Shen, Rev. Mod. Phys. **75**, 473 (2003).
- [14] H.-Y. Kee, H. Doh, T. Grzesiak, J. Phys. Condens. Matter **20**, 255248 (2008).